Chapter 11
Global Illumination

Part 2
Radiosity

Reading:
Angel’s Interactive Computer Graphics (6th ed.)
Section 11.5

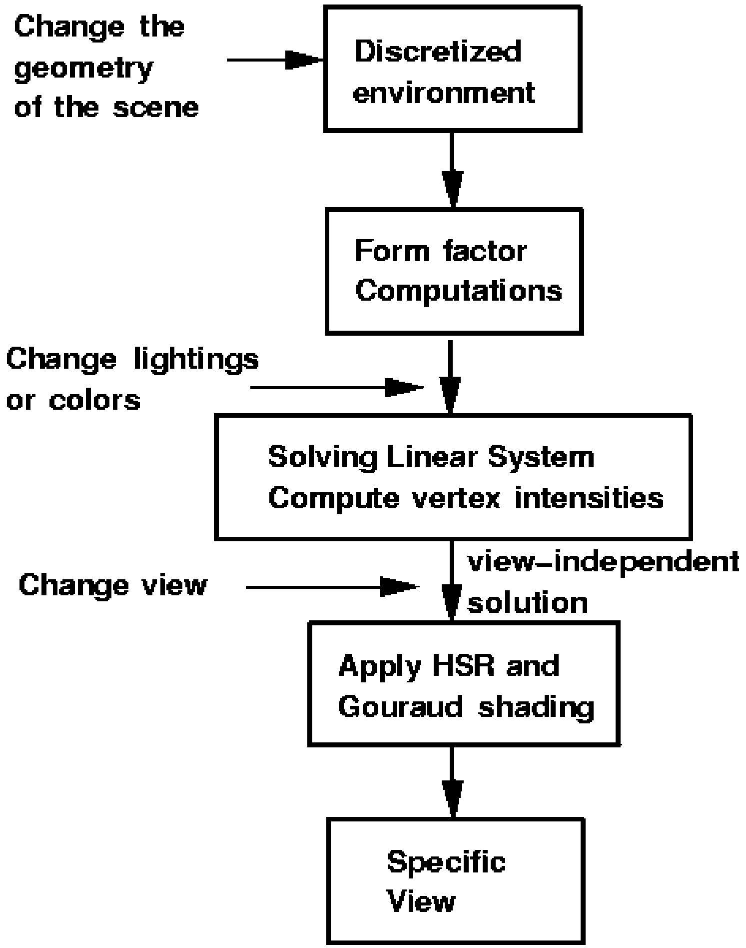
Radiosity -1

- Ray tracing
  - Deals with scenes of perfect specular and transmission patches, and approximates diffuse interaction by ambient term.
  - Scene geometry can be any.
  - Point light source only.
- Radiosity
  - Deals with perfect diffuse interior scenes.
  - Scene geometry is polygonal patches.
  - A single patch reflects light received from every other patches and also emit light if it is a light source.

Radiosity -2

- **Radiosity (cont.)**
  - The diffuse interaction between patches depends on their geometry (distance and orientation).
  - Conservation of light energy leads to an equilibrium in which light intensity is assume to be constant across a patch.
  - A light source is treated like any other surfaces except it possesses an initial (non-zero) radiosity.
  - Shadows are dealt with without add-on procedure, intensity within a shadow can be more effectively handled.
  - Radiosity calculation is view-independent.

Radiosity -3



Change the geometry of the scene

Discretized environment

Form factor Computations

Change lightings or colors

Solving Linear System
Compute vertex intensities

view–independent solution

Change view

Apply HSR and Gouraud shading

Specific View

Radiosity -4
Radiosity Equations

- **Radiosity**
  - **The energy per unit area leaving a surface patch per unit time and is the sum of emitted and the reflected energy**

$$B_i A_i = E_i A_i + \rho_i \sum_{j=1}^{n} B_j F_{ji} A_j$$

$$B_i = E_i + \rho_i \sum_{j=1}^{n} B_j F_{ji} \frac{A_j}{A_i}$$

  - **F_{ji}: the form factor specifying the fraction of energy leaving entire patch j that directly arrives at entire patch i.**

Radiosity -5
Radiosity Equations

- **Reciprocity relation**
  - **For a pair of equally sized emitter and receiver, the fraction of energy emitted by one and received by the other is identical to the refraction of energy going the other way.**

$$A_i F_{ij} = F_{ji} A_j$$

- **With reciprocity relation**

$$B_i = E_i + \rho_i \sum_{j=1}^{n} B_j F_{ij} \quad or \quad B_i - \rho_i \sum_{j=1}^{n} B_j F_{ij} = E_i$$

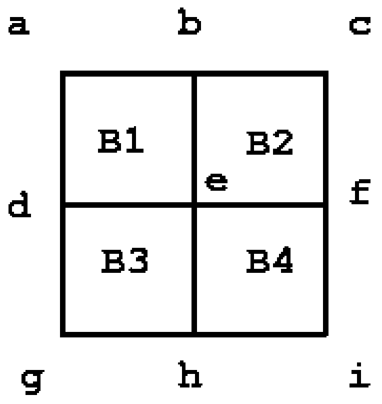
Radiosity -6
Radiosity Equations

$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & -\rho_2 F_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1-\rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

  - **E_i: input illumination to the system**
  - **F_{ij}: form factor, $F_{00}=0$ for plane patch**
  - **ρ_i: reflectivity**
  - **E_i and ρ_i are wavelength dependent; e.g., (R,G,B)**
- **Solution: B_i – a single radiosity value for each patch**

Radiosity -7
Vertex Intensity

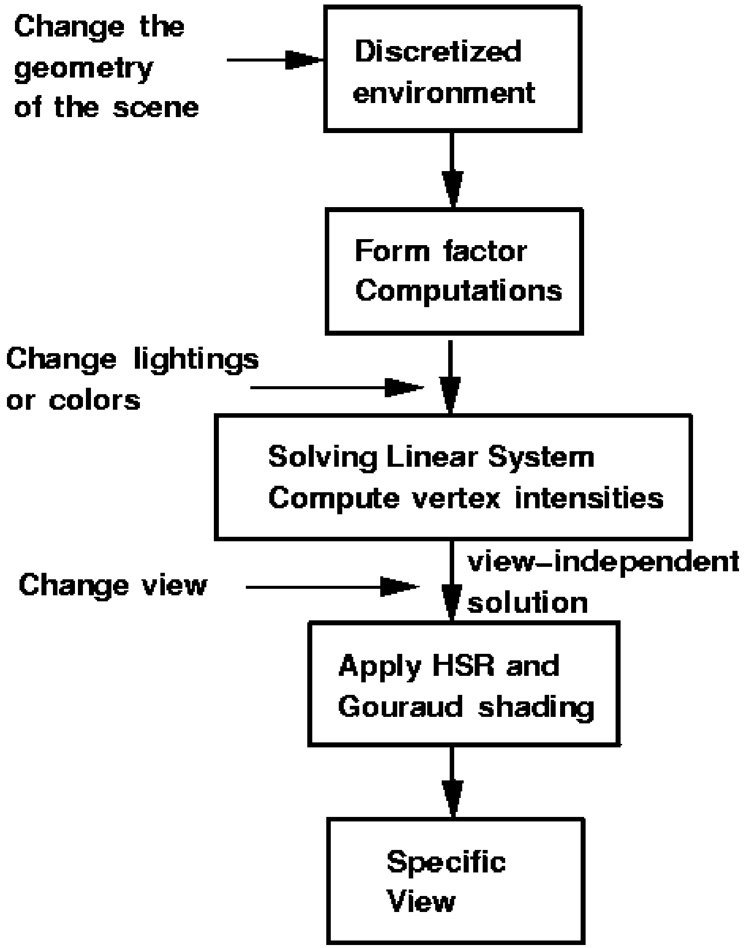
- **Interior vertex e**
  - averaging the radiosities of patches that share e.
- **Boundary vertex b**
  - Find the nearest interior vertex e
  - Apply “vertex’s average = patch’s average”



$$B_e = \frac{1}{4}(B_1 + B_2 + B_3 + B_4)$$

$$\frac{1}{2}(B_b + B_e) = \frac{1}{2}(B_1 + B_2) \Rightarrow B_b$$

Radiosity -8
Rendering



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Radiosity -9
Solution – Gauss-Seidel Method

- Can be solved by any linear solvers
  - Direct methods: Gaussian Elimination, Decomposition
  - Indirect Methods: Jocobi methd, Gauss-Seidel method
  - Large size, better handled by iteration methods.
- Gauss-Seidel iterative method
  - Converges for any strictly diagonally dominant systems from any initial solution.
  - Iteration, initials, termination conditions.

Radiosity -10
Solution – Gauss-Seidel Method

- **The coefficient matrix is strictly diagonally dominant.**
  - The sum of form factors from any patch to other patches must be 1, and the reflectivity is less than 1.
  - Sum of the elements other than the diagonal on any row is less than 1 on the diagonal, if $F_{ii}=0$ for $i=1,2,\ldots,n$ (holds for planar or convex patches).
- **Gauss-Seidel iteration is a gathering process**

Radiosity -11
Cost and accuracy

- Cost
  - Form factor
    - $O(n^2)$ x O(form-factor)
  - Linear system
    - $O(n^3)$
  - Space $O(n^2)$
- Accuracy
    - Accuracy of form factor computation (e.g., Hemicube resolution)
    - Meshing resolution

Progressive Refinement

- **Standard radiosity calculations**
  - Cannot provide a quick or progressive refinement feedback.
- **What do we want?**
  - Find a compromise between the competing demand of interactivity and image quality.
  - Present an initial quickly rendered image to users, then progressively refine in a graceful way towards higher quality in a way that is automatic, continuous and not distracting to users.

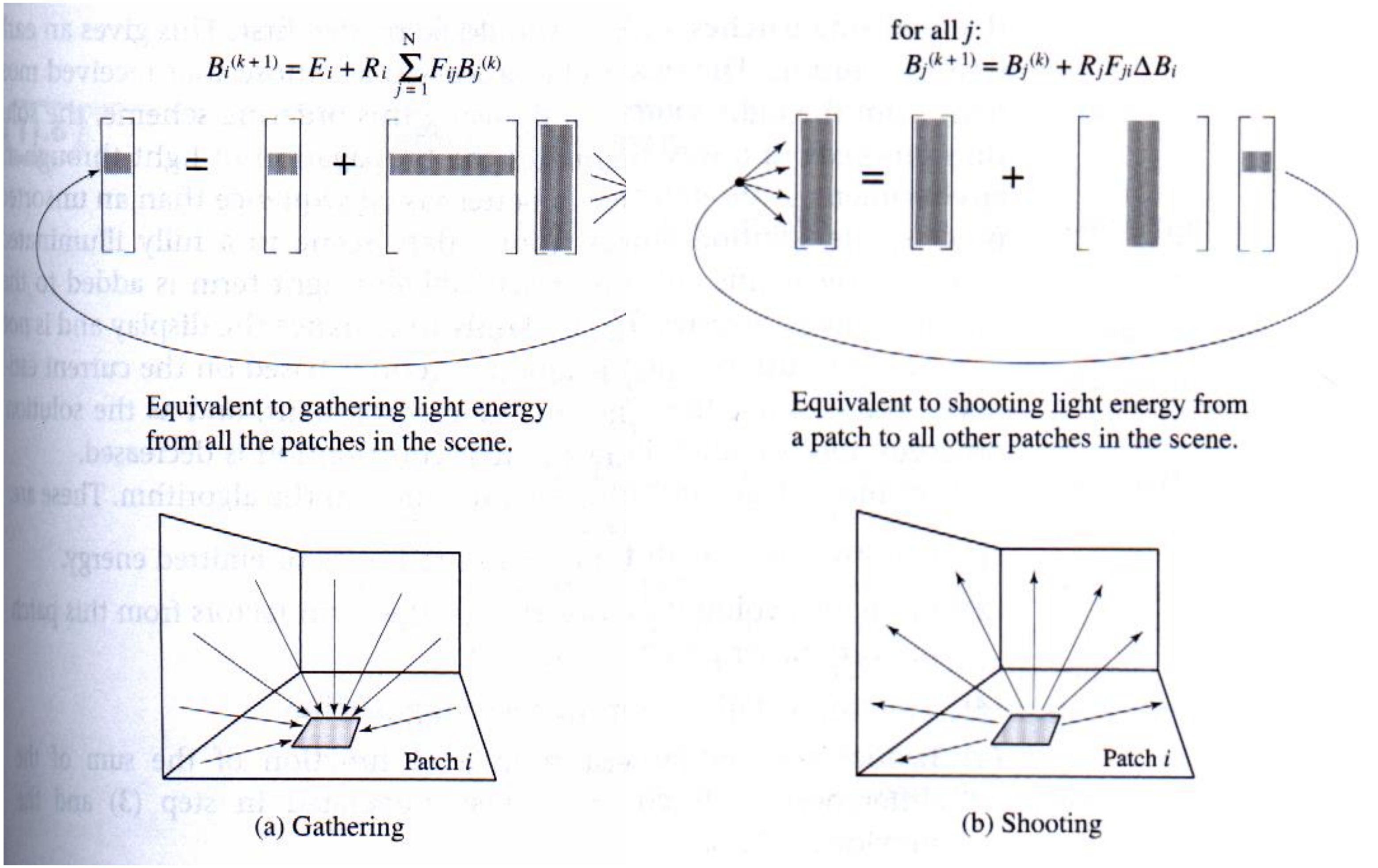
Progressive Refinement

- Another goal
  - Eliminate pre-calculation and storage of the form factors
    - normally n^2; but may be 90% sparse since many patches cannot see each other.
- Progressive refinement procedure
  - The solution is restructured and the form factor evaluation order is optimized so that the convergence is visually graceful.
  - The restructuring enables the radiosity of all patches to be updated at each step and maximum visual difference between steps can be achieved.

Progressive Refinement

- **Standard radiosity calculations**
  - B_i, the solution to a patch, is obtained by **gathering** the current estimate of all other patches.
  - The calculation proceeds on a row-by-row basis and the entire solution is updated for one step.
  - Each patch intensity is updated according to its row position in the radiosity system.
- **Progressive refinement**
  - Enables the update of radiosity for all patches by **shooting** process, where the contribution of patch P_i is distributed to all other patches.

Progressive Refinement



$$B_i^{(k+1)} = E_i + R_i \sum_{j=1}^{N} F_{ij} B_j^{(k)}$$

Equivalent to gathering light energy from all the patches in the scene.

for all j:

$$B_j^{(k+1)} = B_j^{(k)} + R_j F_{ji} \Delta B_i$$

Equivalent to shooting light energy from a patch to all other patches in the scene.

(a) Gathering

(b) Shooting

Gathering vs. Shooting

- **Gathering: for all i**

$$B_i \quad \text{due-to} \quad B_j \quad \text{is} \quad \rho_i B_j F_{ij} \qquad \text{for-all-} j$$

  - **This requires knowing F_{ji} for each (i, j)**

- **Shooting: for i**

$$B_j \quad \text{due-to} \quad B_i \quad \text{is} \quad \rho_j B_i F_{ji} \qquad \text{for-all-j}$$

  - **This requires knowing F_{ji} for each j (using hemicube on each patch j).**
  - **With reciprocity relation, we have**

$$B_j \quad \text{due-to} \quad B_i \quad \text{is} \quad \rho_j B_i F_{ij} \frac{A_i}{A_j} \qquad \text{for-all-j}$$

General Steps

- Select a shooting patch P_i
- Evaluate form factors F_{ij}
- Shooting to P_j for all j <> i
  - Calculate the amount of radiosity shot to P_j from P_i
  - Update ΔB_j
  - Update B_j
- Display the scene
- Check for convergence (if $\Delta B_j < \varepsilon$ for all j)

Pueudo Code

```
Initial $B_i = \Delta B_i = E_i$
Repeat
    Select a shooting patch i
    Calculate $F_{ij}$ for each j
    for each patch j,  j<>I, do {
            $\Delta Rad = \rho_j \Delta B_i F_{ij} A_i / A_j$
            $\Delta B_j = \Delta B_j + \Delta Rad$
            $B_j = B_j + \Delta Rad$
        }
    $\Delta B_i = 0$
    Display the scene using current radiosities
Until convergence
```

Some Notes

- Selecting shooting patch
  - To make the most difference between iterations, we select the patch that has the most energy left to emit.
  - Select the the patch with the largest $A_i \Delta B_i$
- On-the-fly form factor calculation
  - Eliminate pre-calculation and storage of n^2 form factors.
  - Only a single hemicube on the shooting patch and its form factors need to be computed and stored.

Ambient Term Enhancement

- **Early-stage estimates need correction**
  - Images of PR will be very dark in the beginning, especially for areas of the scene that are not directly visible from the light source.
  - Need to add a term that represents an estimate of the effect of subsequent shooting operations, i.e., the effect of propagating all unshot radiosity.
- **Adding correction term**

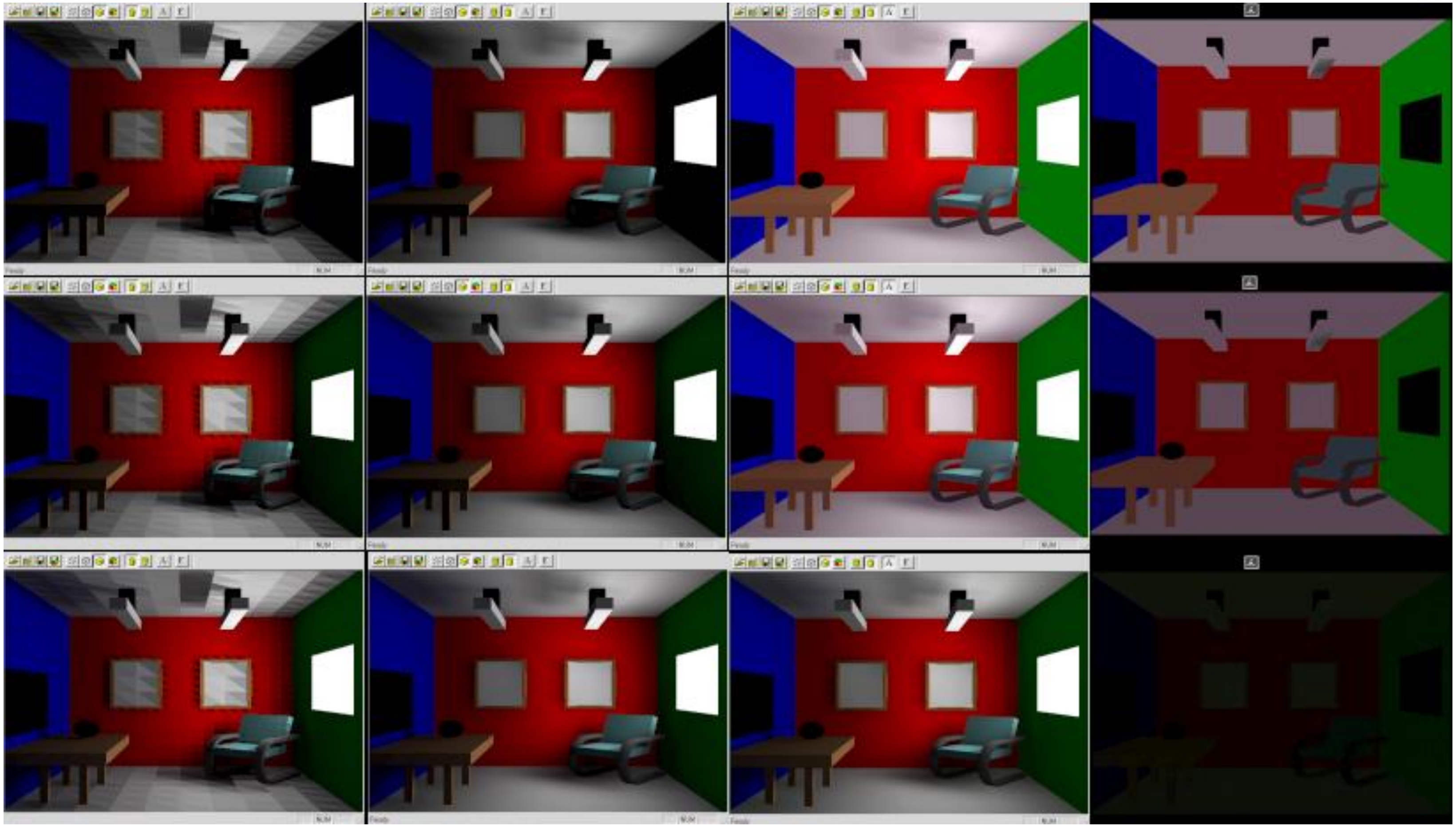
$$B_i' = B_i + \rho_i \quad Ambient$$

How do we estimate *Ambient* ?

- **Total unshot radiosity**
  - **Weighting sum** $U = \sum_{i=1}^{n} \Delta B_i A_i / A$
- **Effect of propagating unshot radiosity**
  - **Average reflectance:** $\rho_{ave} = \sum_{i=1}^{n} \rho_i A_i / A$
  - **Result of the first level of propagation:** $\rho_{ave} U$
  - **Total amount of propagating the unshot radiosity**

$$Ambient = U + \rho_{ave} U + \rho^2{}_{ave} U + \cdots = (1 + \rho_{ave} + \rho^2{}_{ave} + \cdots) U$$

PR Examples



Radiosity -12
Problems

- **Problems due to finite element approach**
  - Shading artifacts
  - Inaccurate shadow boundary
- **Only good for static scenes**
- **Cannot handle**
  - Specular-to-specular propagation
  - Specular-to-diffues propagation
  - Diffuse-to-specular propagation